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Electrophoretic mobility of a "soft particle" with a nonuniformly charged surface layer as a model for cells

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Abstract

The electrophoretic mobility of a colloidal particle with a nonuniformly charged ion-penetrable surface layer in an electrolyte solution (as a model for a biological cell) is calculated for the case in which the surface charge layer consists of two oppositely charged sublayers. It is shown that the mobility and the surface potential (the latter of which is defined as the potential at the front surface of the surface charge layer) generally exhibit different behaviors, in contrast to the case of rigid particles, which is described by Smoluchowski's mobility formula. The mobility in some cases shows a minimum (or a maximum) when plotted as a function of the electrolyte concentration and alters its sign at a certain concentration, which is considered to be an isoelectric point of the mobility with respect to the electrolyte concentration. This isoelectric point is in general different from that of the surface potential.

Keywords: Electrophoretic mobility; Soft particle; Cells

1. Introduction

The surface potential of colloidal particles or membranes plays a fundamental role in their interfacial electric phenomena such as electrostatic interaction between them. In a previous paper [1] we have shown that the surface potential of an ion-penetrable membrane with nonuniform distribution of membrane fixed-charges may reverse its sign as the electrolyte concentration varies at constant pH. In other words there exists an "isoelectric point" with respect to change in electrolyte concentration. In the previous paper [1] we considered the case in which the membrane consists of two oppositely charged layers; acidic groups are distributed in the outer layer while basic groups in the inner layer. The reason for the sign reversal of the surface potential can be explained as follows. It is the fixed-charges located trough the depth (form the membrane surface) of the order of the Debye length $1/\kappa$ (κ being given later in eq. (16)) that mainly contribute to the membrane surface potential. At high electrolyte concentrations, $1/\kappa$ is small so that the sign of the surface potential is determined mainly by the sign of the fixed-charges present in the outer layer. On the other hand, at low electrolyte concentrations, $1/\kappa$ is large and

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the contribution from the fixed charges in the inner layer becomes appreciable. Thus, if the inner layer is sufficiently highly charged, then at low electrolyte concentrations the sign of the surface potential may coincide with the sign of the fixed-charges in the inner layer. The results obtained in the previous paper [1] may be extended to colloidal particles covered with ion-penetrable surface charge layers consisting of charged polymers, such as biological cells, if the surface charge layer is considered to be an ion-penetrable membrane and the surface potential is defined as the potential at the front surface of the surface charge layer.

One of the common methods for determining the surface potential of colloidal particles is electrophoresis. This method measures the electrophoretic mobility of colloidal particles moving in an electrolyte solution. The mobility is then converted to their zeta potential via Smoluchowski's formula [2]. When the electophoresis of a colloidal particle obeys the Smoluchowski's theory, the zeta potential and the mobility are proportional to each other. If, further, the slipping plane coincides with the particle surface, then the surface potential is equal to the zeta potential so that the mobility and the surface potential show the same behavior (such as their dependence on electrolyte concentration).

However, recent theoretical studies [3–9] reveal that colloidal particles covered with surface charge layers, which we call "soft particles" (Fig. 1), exhibit different electrophoretic behavior from "rigid particles". In particular, our previous studies [8,9] show that the zeta potential of a soft particle loses its meaning.

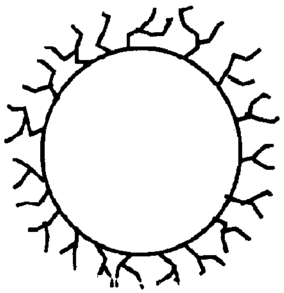

Fig. 1. Schematic representation of a soft particle. The particle is covered by an ion-penetrable surface charge layer consisting of charged polymers.

The purpose of the present paper is to derive an approximate mobility formula for the case where the surface charge layer consists of two oppositely charged sublayers, which is the same system as that considered in the previous paper [1], on the basis of linearized Poisson–Boltzmann equations. Then we discuss the dependence of the mobility on the electrolyte concentration.

2. Theory

Consider a colloidal particle moving with velocity U in a liquid containing a symmetrical electrolyte of valency ν and bulk concentration n in an applied electric field E. Imagine that the colloidal particle is covered with an ion-penetrable surface charge layer of thickness d_c consisting of charged polymers. The particle radius is much larger than $1/\kappa$ (κ being the Debye–Hückel parameter) so that the particle surface can be assumed to be planar and the applied field E to be parallel to the surface. Also we assume that d_c is much greater than $1/\kappa$. We take an x-axis perpendicular to the particle surface with its origin at the front surface of the surface charge layer (Fig. 2).

The flow velocity of the liquid $u(x)$ (relative to the particle), which is parallel to the particle surface, is described by the following Navier–Stokes equations:

$$\eta\frac{d^2u}{dx^2}+\rho_{el}(x)E=0, \quad x>0, \tag{1}$$

$$\eta\frac{d^2u}{dx^2}-\gamma u(x)+\rho_{el}(x)E=0, \quad -d_s<x<0, \tag{2}$$

where η is the viscosity of the liquid, γ the frictional coefficient of the surface charge layer and $\rho_{el}(x)$ the charge density resulting from electrolyte ions at position x. The term $-\gamma u$ arises from the drag force exerted by the polymer segments in the surface charge layer.

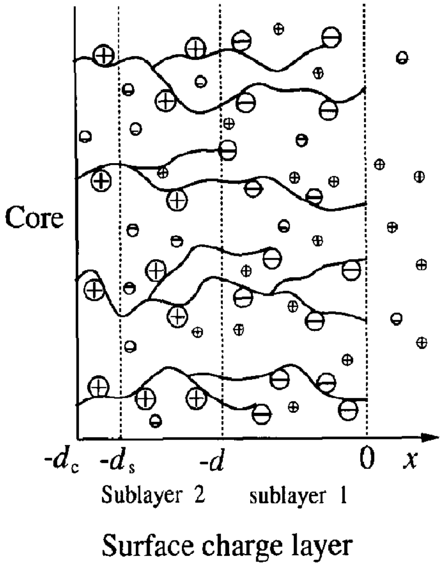

Fig. 2. Schematic representation of a soft surface covered by an ion-penetrable surface charge layer consisting of two oppositely charged sublayers 1 and 2. Fixed charges are represnted by large circles with plus or minus signs, while electrolyte ions are by large circles with plus or minus signs. The total thickness of the surface charge layer is d_c. The thickness of the outer layer (sublayer 1) is d. The density and valency of fixed charged groups distributed in sublayer i are, respectively, N_i and z_i ($i = 1, 2$). The slipping plane is located at $x = -d_s$($d_c \geq d_s > d > 0$). It is assumed that $\kappa d_c \geq \kappa d_s \gg 1$. The electric potential at the front surface of the surface charge layer is defined as the surface potential.

The boundary conditions for $u(x)$ are then

$$u(-d_s) = 0, \tag{3}$$

$$u(-0) = u(+0), \tag{4}$$

$$\left.\frac{du}{dx}\right|_{x=-0} = \left.\frac{du}{dx}\right|_{x=+0}, \tag{5}$$

$$u(x) \to -U \quad \text{as} \quad x \to +\infty. \tag{6}$$

Equation (3) expresses the boundary condition that the slipping plane, at which $u(x)$ is zero, is assumed to be located at $x = -d_s$ ($0 \leq d_s \leq d_c$). Equations (5) and (6) are the continuity conditions of $u(x)$ and du/dx at $x = 0$. Equation (6) states that the liquid velocity in the bulk solution phase equals the electophoretic velocity, but with opposite sign.

In order to calculate the mobility, one needs to know the distribution of the electric potential $\psi(x)$ within the surface charge layer. The potential distribution $\psi(x)$ is given by the Poisson equations, viz.,

$$\frac{d^2\psi}{dx^2} = -\frac{\rho_{el}(x)}{\varepsilon_r \varepsilon_0}, \quad x > 0 \tag{7}$$

$$\frac{d^2\psi}{dx^2} = -\frac{\rho el(x) + \rho_{fix}(x)}{\varepsilon_r \varepsilon_0}, \quad -d_c < x < 0 \tag{8}$$

where $\rho_{el}(x)$ and $\rho_{fix}(x)$ are, respectively, the charge density resulting from electrolyte ions and the density of fixed-charges in the surface charge layer, ε_r the relative permittivity of the electrolyte solution and ε_0 the permittivity of a vacuum. Since we have assumed that the particle surface is planar and the applied electric field is parallel to it, the potential distribution $\psi(x)$ is not perturbed by the applied field and thus it is at equilibrium. We assume a Boltzmann distribution for electrolyte ions. Then, eqs. (7) and (8) become the following Poisson–Boltzmann equations:

$$\frac{d^2\psi}{dx^2} = \frac{2n\nu e}{\varepsilon_r \varepsilon_0} \sinh\frac{\nu e\psi}{kT}, \quad x > 0, \tag{9}$$

$$\frac{d^2\psi}{dx^2} = \frac{2n\nu e}{\varepsilon_r \varepsilon_0} \sinh\frac{\nu e\psi}{kT} - \frac{\rho_{fix}(x)}{\varepsilon_r \varepsilon_0}, \quad -\mathrm{d}_c < x < 0, \tag{10}$$

where e is the elementary electric charge, k Boltzmann's constant and T the absolute temperature. The boundary conditions for $\psi(x)$ are

$$\psi(-0) = \psi(+0), \tag{11}$$

$$\left.\frac{d\psi}{dx}\right|_{x=-0} = \left.\frac{d\psi}{dx}\right|_{x=+0}, \tag{12}$$

$$\left.\frac{d\psi}{dx}\right|_{x=-d_s} = 0. \tag{13}$$

Equations (11) and (12) are the continuity conditions of ψ and of $d\psi/dx$ at $x = 0$. Equation (13) states that the surface of the particle core ($x = -d_c$) is uncharged and the electric field within the particle core is negligible.

On the basis of eqs. (1), (2), (7) and (8) subject to boundary conditions (3)–(6), we derive the following approximate formula for the electrophoretic mobility (see eq. (22) in Ref. [9])

$$\mu = \frac{\varepsilon_r\varepsilon_0}{\eta}\lambda\int_{-\infty}^{0}\psi(x)\,\mathrm{e}^{\lambda x}\,\mathrm{d}x + \frac{1}{\eta\lambda}\int_{-\infty}^{0}\rho_{\mathrm{fix}}(x)\,\mathrm{e}^{\lambda x}\,\mathrm{d}x, \tag{14}$$

with

$$\lambda = (\gamma/\eta)^{1/2} \tag{15}$$

Equation (14) is applicable when $d_c > d_s \gg 1/\kappa$, $1/\lambda$, where κ is the Debye–Hückel parameter defined by

$$\kappa = \left(\frac{2n\nu^2e^2}{\varepsilon_r\varepsilon_0 kT}\right)^{1/2}. \tag{16}$$

Thus, the lower limit $-d_s$ of the integral over the surface charge layer in eq. (14) has been replaced by $-\infty$.

Further, we assume that $\psi(x)$ is small enough to obey the following linearized form of Poisson–Boltzmann equations:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = \kappa^2\psi, \qquad x > 0, \tag{17}$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = \kappa^2\psi - \frac{\rho_{\mathrm{fix}}(x)}{\varepsilon_r\varepsilon_0}, \qquad -d_c < x < 0. \tag{18}$$

Consider the case where the surface charge layer consists of two sublayers (Fig. 2). The outer sublayer (sublayer 1) carries ionized groups of valency z_1 and number density N_1, while the inner sublayer (sublayer 2) carries ionized groups of valency z_2 and number density N_2. Let the thickness of layer 1 be d. Then, we have

$$\rho_{\mathrm{fix}}(x) = \rho_1 = ez_1N_1, \qquad -d \le x \le 0, \tag{19}$$

$$\rho_{\mathrm{fix}}(x) = \rho_2 = ez_2N_2, \qquad x \le -d, \tag{20}$$

where ρ_1 and ρ_2 are, respectively, the charge densities of fixed charges in sublayers 1 and 2. Since we have assumed that $\kappa d_s \gg 1$, then d_s can practically be set equal to infinity for mathematical convenience. Thus eq. (13) may be replaced by the condition that in the deep inside of sublayer 2, the potential derivative (or the electric field) must be zero. The potential in this region is equal to the Donnan potential in sublayer 2, $\psi_{\mathrm{DON},2}$ (given later by eq. (25)). The solution to eqs. (17) and (18) subject to the continuity conditions at $x = 0$ (eqs. (11) and (12)) and similar continuity conditions at $x = -d$ is given by

$$\psi(x) = \tfrac{1}{2}\psi_{\mathrm{DON},1}[\mathrm{e}^{-\kappa x} - \mathrm{e}^{-\kappa(x+d)}] + \tfrac{1}{2}\psi_{\mathrm{DON},2}\,\mathrm{e}^{-\kappa(x+d)}, \qquad x>0, \tag{21}$$

$$\psi(x) = \psi_{\mathrm{DON},1}[1 - \tfrac{1}{2}\{\mathrm{e}^{\kappa x} + \mathrm{e}^{-\kappa(x+d)}\}] + \tfrac{1}{2}\psi_{\mathrm{DON},2}\,\mathrm{e}^{-\kappa(x+d)}, \qquad -d<x<0, \tag{22}$$

$$\psi(x) = \tfrac{1}{2}\psi_{\mathrm{DON},1}[\mathrm{e}^{\kappa(x+d)} - \mathrm{e}^{\kappa x}] + \psi_{\mathrm{DON},2}[1 - \tfrac{1}{2}\,\mathrm{e}^{\kappa(x+d)}], \qquad x< -d, \tag{23}$$

with

$$\psi_{\mathrm{DON},1} = \frac{ez_1N_1}{\varepsilon_r\varepsilon_0\kappa^2}, \tag{24}$$

$$\psi_{\mathrm{DON},2} = \frac{ez_2N_2}{\varepsilon_r\varepsilon_0\kappa^2}, \tag{25}$$

where $\psi_{\mathrm{DON},1}$ and $\psi_{\mathrm{DON},2}$ are, respectively, the Donnan potentials of sublayers 1 and 2. Equations (24) and (25) are correct to the order of N_1 and N_2, respectively. These expressions are consistent to the linearized Poisson–Boltzmann eqs. (17) and (18). The surface potential, which we define as the potential at $x = 0$, i.e., at the front surface of the surface charge layer, is calculated by setting $x = 0$ in eq. (21) or (22), viz.,

$$\psi(0) = \tfrac{1}{2}\psi_{\mathrm{DON},1}(1 - \mathrm{e}^{-\kappa d}) + \tfrac{1}{2}\psi_{\mathrm{DON},2}\,\mathrm{e}^{-\kappa d} = \frac{e}{2\varepsilon_r\varepsilon_0\kappa^2}[z_1N_1(1 - \mathrm{e}^{-\kappa d}) + z_2N_2\,\mathrm{e}^{-\kappa d})], \tag{26}$$

Substitution of eq. (22) and (23) into eq. (14) yields

$$\mu = \frac{e}{\eta\lambda^2}\left[z_1N_1\left\{1 + \left(\frac{\lambda}{\kappa}\right)^2\frac{1+\lambda/2\kappa}{1+\lambda/\kappa}\right\} + (z_1N_1 - z_2N_2)\left\{\left(\frac{\lambda}{\kappa}\right)^2\frac{\lambda}{2(\kappa-\lambda)}\mathrm{e}^{-\kappa d} - \frac{\kappa^2}{\kappa^2-\lambda^2}\mathrm{e}^{-\lambda d}\right\}\right]. \tag{27}$$

This is the required result. If the surface charge layer is uniformly charged, i.e., $z_1N_1 = z_2N_2$ ($\rho_1 = \rho_2$), then eq. (27) reduces to a formula derived in the previous paper (eq. (39) in Ref. [9]).

3. Results and discussion

We have derived an expression for the electrophoretic mobility of a particle covered with an ion-penetrable surface charge layer. Such a particle (which we call a "soft particle") serves as a model for biological cells (eq. (27)). This mobility expression applies to the case where the surface charge layer consists of two sublayers 1 and 2, carrying fixed-charges of uniform densities ez_1N_1 and ez_2N_2, respectively. Consider the case where these two layers are oppositely charged, that is, the outer sublayer (sublayer 1) is negatively charged ($z_1 < 0$), whereas the inner sublayer (sublayer 2) is positively charged ($z_2 > 0$).

Some results of the calculation based on eq. (27) are given in Figs. 3–5. The values of parameters employed are $z_1N_1 = -0.2\ M$ and $z_2N_2 = +0.1\ M$. Figures 3 and 4, respectively, depict the electrophoretic mobility μ in a 1:1 symmetrical electrolyte ($\nu = 1$) as a function of the electrolyte concentration n for several values of $1/\lambda$ with d kept constant at 1 nm and those for several values of d with $1/\lambda$ kept constant at 1 nm.

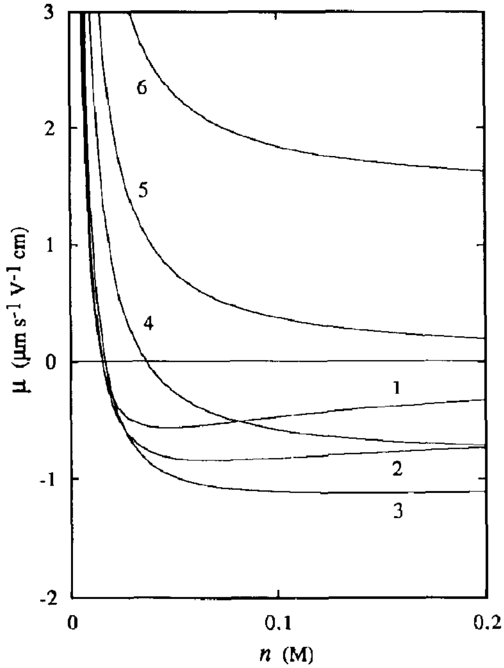

Fig. 3. The electrophoretic mobility μ as a function of electrolyte concentration n for various values fo $1/\lambda$ at $d = 1$nm, calculated via eq. (27) with $\nu = 1$, $z_1N_1 = -0.2\ M$, $z_2N_2 = +0.1\ M$, $\eta = 0.89 \times 10^{-3}$ N m^{-2}s, $\varepsilon_r = 78.5$ and $T = 298$ K. Curves: (1) $1/\lambda = 0$, (2) $1/\lambda = 0.5$ nm, (3) $1/\lambda = 1$ nm, (4) $1/\lambda = 2$ nm, (5) $1/\lambda = 2.5$ nm and (6) $1/\lambda = 3$ nm. Curve 1 ($1/\lambda = 0$) corresponds to Smoluchowski's equation, eq. (33). Only for this case the mobility and the surface potential $\psi(0)$ are proportional and curve 1 shows also the dependence of $\psi(0)$ on n.

In order to analyze the dependence of the mobility on the electrolyte concentration, it is convenient to rewrite the mobility expression (27) as the sum of the contributions from sublayer 1 and sublayer 2, which we denote by μ_1 and μ_2, respectively, viz.,

$$\mu = \mu_1 + \mu_2, \tag{28}$$

with

$$\mu_1 = \frac{ez_1N_1}{\eta\lambda^2}\left[1 + \left(\frac{\lambda}{\kappa}\right)^2 \left\{\frac{1 + \lambda/2\kappa}{1 + \lambda/\kappa} + \frac{\lambda}{2(\kappa - \lambda)}\mathrm{e}^{-\kappa d}\right\} - \frac{\kappa^2}{\kappa^2 - \lambda^2}\mathrm{e}^{-\lambda d}\right], \tag{29}$$

$$\mu_2 = -\frac{ez_2N_2}{\eta\lambda^2}\left[\left(\frac{\lambda}{\kappa}\right)^2 \frac{\lambda}{2(\kappa - \lambda)}\mathrm{e}^{-\kappa d} - \frac{\kappa^2}{\kappa^2 - \lambda^2}\mathrm{e}^{-\lambda d}\right], \tag{30}$$

In order to analyze the behavior of the mobility, let us consider several limiting cases.

In the limit of large λ (curve 1 in Fig. 3),

$$\mu_1 \to \frac{ez_1N_1}{2\eta\kappa^2}(1 - \mathrm{e}^{-\kappa d}) \quad \text{and} \quad \mu_2 \to \frac{ez_2N_2}{2\eta\kappa^2}\mathrm{e}^{-\kappa d}, \tag{31}$$

and thus

$$\mu = \frac{e}{2\eta\kappa^2}\left[z_1N_1(1 - \mathrm{e}^{-\kappa d}) + z_2N_2\,\mathrm{e}^{-\kappa d}\right]. \tag{32}$$

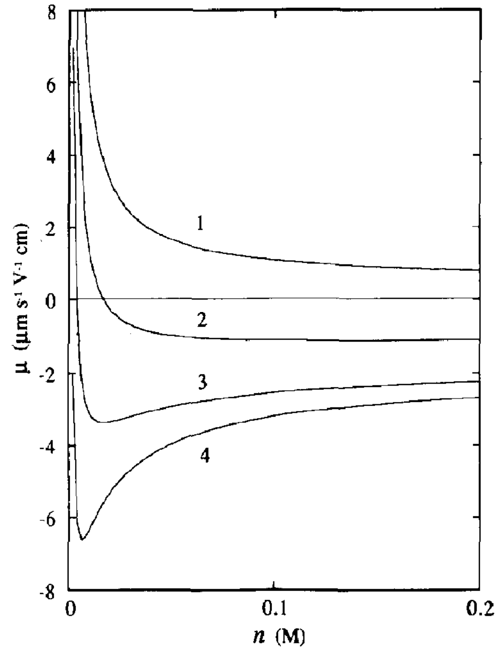

Fig. 4. The electrophoretic mobility μ as a function of electrolyte concentration n for various values of d at $1/\lambda = 1$ nm, calculated from eq. (27) with $\nu = 1$, $z_1N_1 = -0.2$ M, $z_2N_2 = +0.1$ M, $\eta = 0.89\times10^{-3}$ N m^{-2} s, $\varepsilon_r = 78.5$ and $T = 298$ K. Curves: (1) $d = 0.2$ nm, (2) $d = 1$ nm, (3) $d = 2$ nm, and (4) $d = 3$ nm.

This case corresponds to the situation in which the frictional force exerted by the polymer segments in the surface charge layer upon the liquid is very large (i.e., frictional coefficient λ is very large) so that the liquid flow disappears in the surface charge layer. It is easily seen that eq. (31) can be rewritten as

$$\mu = \frac{\varepsilon_r\varepsilon_0}{\eta}\psi(0), \tag{33}$$

where $\psi(0)$ is the surface potential, given by eq. (26). Equation (33) is the Smoluchowski result [2] when the slipping plane is shifted to the front surface of the surface charge layer, $x = 0$. Thus, curve 1 ($1/\lambda = 0$) in Fig. 3 shows also the dependence of the surface potential $\psi(0)$. In other words, in the limit of $\lambda \to \infty$ ($1/\lambda \to 0$), the mobility and the surface potential $\psi(0)$ exhibit the same behavior. However, as the value of $1/\lambda$ increases, deviation of the mobility behavior from that of the surface potential increases, as is seen in Fig. 3. In particular, curves 5 and 6 in Fig. 3 and curve 1 in Fig. 4 show neither minimum nor zero, as will be discussed agian later. Thus, the behavior of the mobility μ is in general different from that of the surface potential $\psi(0)$, except in the Smoluchowski's limit $\lambda \to \infty$. Also it must be mentioned that at small $1/\lambda$ the position of the mobility zero is not very sensitive to the value of $1/\lambda$ (see curves 1–3).

In the opposite limit, i.e., at small λ,

$$\mu_1 \to 0 \quad \text{and} \quad \mu_2 \to \frac{ez_2N_2}{\eta\lambda^2}\left[1 + \left(\frac{\lambda}{\kappa}\right)^2\right]. \tag{34}$$

Thus, the mobility is determined by the contribution from the inner sublayer (sublayer 2). This is because decreased frictional forces exerted from the surface charge layer on the liquid causes a relative increase of the contribution from fixed charges located in the deep inside of the surface charge layer (i.e., from the inner sublayer). This is the reason why the mobility shows neither minimum nor zero (curves 5 and 6 in Fig. 3 and curve 1 in Fig. 4).

Variation of the mobility with electrolyte concentration n is caused not only by the shielding effects of electrolyte ions but also by the change in the relative contributions from the fixed-charges in layer 1 and from those in layer 2. The latter effects correspond to the fact that fixed-charges located through the depth of around $1/\kappa$ mainly contributes to the mobility.

At low electrolyte concentrations n (i.e., large $1/\kappa$), because of the increase of the contribution from the positively charged inner sublayer (sublayer 2), the mobility becomes positive. Indeed, in the limit of small n (or large $1/\kappa$),

$$\mu_1 \to 0 \quad \text{and} \quad \mu_2 \to \frac{ez_2N_2}{2\eta\kappa^2}, \tag{35}$$

so that the sign of the mobility always coincides with the sign of sublayer 2 (i.e., the sign of z_2).

Next consider the limiting case of large n (small $1/\kappa$). In this limit,

$$\mu_1 \to \frac{ez_1N_1}{\eta\lambda^2}(1 - e^{-\lambda d}) \quad \text{and} \quad \mu_2 \to \frac{ez_2N_2}{\eta\lambda^2}e^{-\lambda d}, \tag{36}$$

so that eq. (27) tends to

$$\mu = \frac{e}{\eta\lambda^2}\left[z_1N_1(1-\mathrm{e}^{-\lambda d}) + z_2N_2\,\mathrm{e}^{-\lambda d}\right]. \qquad (37)$$

Note that except for the case of $\lambda \to \infty$, which corresponds to Smoluchowski's equation, eq. (33), the mobility tends to a non-zero limiting value. The fact that the mobility shows a non-zero value in the limit of high electrolyte concentrations, in contrast to Smoluchowski's formula, has been mentioned in the previous paper [9]. Only for the case of $\lambda \to \infty$ $(1/\lambda \to 0)$, the mobility tends to zero as n increases. Also note that the limiting mobility value for $1/\lambda \neq 0$ may become positive or negative. If

$$\frac{|z_2|N_2}{|z_1|N_1} < \mathrm{e}^{\lambda d} - 1, \qquad (38)$$

then μ becomes negative (curves 2–4 in Fig. 3 and curves 2–4 in Fig. 4), whereas if

$$\frac{|z_2|N_2}{|z_1|N_1} > \mathrm{e}^{\lambda d} - 1 \qquad (39)$$

then μ becomes positive (curves 5 and 6 in Fig. 3 and curve 1 in Fig. 4). It can easily be shown that if the mobility has a zero, then the mobility also has a minimum. The criterion for the existence of the minimum is thus given by the inequality (38). That is, when eq. (38) holds, then the mobility has a minimum. The reason why the mobility may have a zero and a minimum can be explained via Fig. 5, which separately shows the contribution from the sublayer 1(μ_1) and that from sublayer 2 (μ_2) as well as the total mobility value μ. If sublayer 1 is positively charged and sublayer 2 negatively charged, then the mobility shows a maximum when eq. (38) holds.

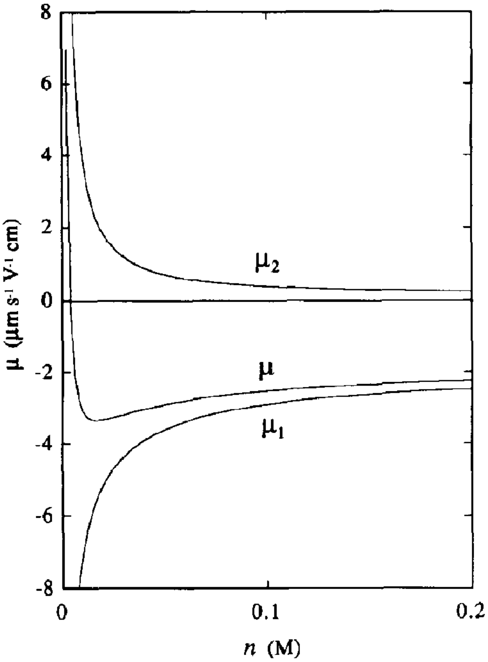

Fig. 5. Contribution from the outer sublayer (layer 1), μ_1, and that from the inner sublayer (layer 2) μ_2, as well as the total mobility $\mu = \mu_1 + \mu_2$ as functions of of electrolyte concentration n at $1/\lambda = 1$ nm and at $d = 2$ nm, calculated via eqs. (27)–(30) with $\nu = 1$, $z_1N_1 = -0.2$ M, $z_2N_2 = +0.1$ M, $\eta = 0.89 \times 10^{-3}$ N m^{-2} s, $\varepsilon_r = 78.5$ and $T = 298$ K. The curve for μ is the same as curve 3 in Fig. 4.

Kawahata et al. [10] measured the electrophoretic mobility of human erythrocytes in electrolyte solutions of various concentrations. Nagahama et al. [10] have recently carried out similar measurements for guinea-pig polymorphonuclear leukocytes. Both of these studies demonstrate that these cells exhibit a negative mobility in usual physiological conditions but as the electrolyte concentration decreases the mobility reverses its sign in some cases. This behavior can be well explained by the present theory, suggesting that the deep region of the surface charge layer of these cells is positively charged. The mobility formula derived in the present paper, eq. (27) will thus be useful in analyzing the electrophoretic mobility of biological cells and provide us with information on the charge distribution in their surface region.

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